STEPENOVANJE

Proizvod $\underset{n-puta}{a \cdot a \cdot ... \cdot a} = a^n$ naziva se n-tim stepenom broja. Ako je $a \in R$, $a \neq 0$ i neka je $n \in N$

Po definiciji je:

1) $a^0 = 1 \rightarrow$ primer: $5^0 = 1$, $(-3)^0 = 1$, $\left(\dfrac{4}{7}\right)^0 = 1$

2) $a^{-n} = \dfrac{1}{a^n} \rightarrow$ primer: $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$, $5^{-3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

Još važe sledeća pravila:

3) $a^m \cdot a^n = a^{m+n}$ $\rightarrow$ primer: $3^2 \cdot 3^5 = 3^{2+5} = 3^7$

4) $a^m : a^n = a^{m-n}$ $\rightarrow$ primer: $7^{10} : 7^6 = 7^{10-6} = 7^4$

5) $(a^m)^n = a^{m \cdot n}$ $\rightarrow$ primer: $(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$

6) $(a \cdot b) = a^n \cdot b^n$ $\rightarrow$ primer: $(12 \cdot 11)^5 = 12^5 \cdot 11^2$

7) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $\rightarrow$ primer $\left(\dfrac{7}{4}\right)^2 = \dfrac{7^2}{4^2}$

8) $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$ $\rightarrow$ primer $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{3^2}{2^2} = \dfrac{9}{4}$

O čemu treba voditi računa?

Treba paziti na zapis: $(-5)^2 = (-5)(-5) = 25$, dok $-5^2 = -5 \cdot 5 = -25$. Uopšteno važi:

$(-a)^{paran} = a^{paran}$

$(-a)^{neparan} = -a^{neparan}$

Dakle, paran izložilac ''uništi'' minus.

ZADACI

1) Izračunati: $\dfrac{(2^7 : 2^5) \cdot 2^3}{2^4 : 2^2}$

$$\frac{(2^7 : 2^5) \cdot 2^3}{2^4 : 2^2} = \frac{2^{7-5} \cdot 2^3}{2^{4-2}} = \frac{2^2 \cdot 2^3}{2^2} = \frac{2^{2+3}}{2^2} = \frac{2^5}{2^2} = 2^{5-2} = 2^3 = 8$$

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2) Izračunati: $\dfrac{3^5 \cdot 9^3}{27^2 \cdot 3}$

$$\frac{3^5 \cdot 9^3}{27^2 \cdot 3} = \frac{3^5 \cdot (3^2)^3}{(3^3)^2 \cdot 3^1} = \frac{3^5 \cdot \cancel{3^6}}{\cancel{3^6} \cdot 3^1} = \frac{3^5}{3^1} = 3^{5-1} = 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

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3) Izračunati: $\dfrac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3} =$

$$\frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3} = \frac{x^{12} \cdot x^3 : x^5}{(x^{5-2})^3} = \frac{x^{12+3-5}}{(x^3)^3} = \frac{x^{10}}{x^9} = x^{10-9} = x^1 = x$$

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4) Izračunati: $\dfrac{3^{n+1} \cdot 3^{n+2}}{3^{2n+4}}$

$$\frac{3^{n+1} \cdot 3^{n+2}}{3^{2n+4}} = \frac{3^{n+1+n+2}}{3^{2n+4}} = \frac{3^{2n+3}}{3^{2n+4}} = \text{ Pazi pa zagrade zbog minusa}$$

$$= 3^{(2n+3)-(2n+4)} = 3^{2n+3-2n-4} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

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5) **Izračunati** $\quad 0,5^{-1} + 0,25^{-2} + 0,125^{-3} + 0,0625^{-4}$

$$0,5^{-1} + 0,25^{-2} + 0,125^{-3} + 0,0625^{-4} =$$

$$\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{8}\right)^{-3} + \left(\frac{1}{16}\right)^{-4} =$$

$$\left(\frac{2}{1}\right)^{1} + \left(\frac{4}{2}\right)^{2} + \left(\frac{8}{1}\right)^{3} + \left(\frac{16}{1}\right)^{4} =$$

$$2^1 + 4^2 + 8^3 + 16^4 = 2 + 16 + 512 + 65536 = 66066$$

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6) **Izračunati** $\quad 1^{-1} + 2^{-2} + 3^{-3} + (-1)^{-1} + (-2)^{-2} + (-3)^{-3}$

$$1^{-1} + 2^{-2} + 3^{-3} + (-1)^{-1} + (-2)^{-2} + (-3)^{-3} =$$

$$\frac{1}{1} + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{(-1)^1} + \frac{1}{(-2)^2} + \frac{1}{(-3)^3} =$$

$$1 + \frac{1}{4} + \frac{1}{27} - 1 + \frac{1}{4} - \frac{1}{27} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

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7) Ako je $\quad a = 5^3 \cdot \left(\frac{1}{4}\right)^{-4} \cdot \left(\frac{3}{2}\right)^2$ i $\quad b = 10^3 \left(\frac{5}{3}\right)^{-2}$ nadji $a \cdot b^{-1}$

$$a = 5^3 \cdot \left(\frac{1}{4}\right)^{-4} \cdot \left(\frac{3}{2}\right)^2 = 5^3 \cdot \left(\frac{4}{1}\right)^4 \cdot \frac{3^2}{2^2} = \frac{5^3 \cdot 4^4 \cdot 3^2}{2^2}$$

$$= \frac{5^3 \cdot (2^2)^4 \cdot 3^2}{2^2} = 5^3 \cdot (2^2)^3 \cdot 3^2 = 5^3 \cdot 2^6 \cdot 3^2$$

$$b = 10^3 \cdot \left(\frac{5}{3}\right)^{-2} = 10^3 \cdot \left(\frac{3}{5}\right)^2 = \frac{10^3 \cdot 3^2}{5^2} = \frac{(5 \cdot 2)^3 \cdot 3^2}{5^2} = \frac{5^3 \cdot 2^3 \cdot 3^2}{5^2} = 5 \cdot 2^3 \cdot 3^2$$

Konačno: $\quad$ izračunati i $\quad a \cdot b^{-1}$

$$a \cdot b^{-1} = 5^3 \cdot 2^6 \cdot 3^2 \cdot \frac{1}{5 \cdot 2^3 \cdot 3^2} = 5^2 \cdot 2^3 = 25 \cdot 8 = 200$$

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8) Izračunati $\left(\left(\dfrac{5x^{-5}}{2y^{-2}}\right)^{-2}\cdot\left(\dfrac{y^{-1}}{5x^{-1}}\right)^{-3}\right):10x^2y^{-3}$

$$\left(\left(\frac{5x^{-5}}{2y^{-2}}\right)^{-2}\cdot\left(\frac{y^{-1}}{5x^{-1}}\right)^{-3}\right):10x^2y^{-3} =$$

$$\left(\frac{5^{-2}\cdot x^{10}}{2^{-2}\cdot y^4}\cdot\frac{y^3}{5^{-3}\cdot x^3}\right):10x^2y^{-3} =$$

$$(5^{-2+3}\cdot x^{10-3}\cdot y^{3-4}\cdot 2^2):10x^2y^{-3} =$$

$$(5^1\cdot x^7\cdot y^{-1}\cdot 4):10x^2y^{-3} =$$

$$\frac{20}{10}x^{7-2}y^{-1-(-3)} = 2x^5y^{-1+3} = 2x^5y^2$$

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9) Ako je $\quad 10^x = \dfrac{\dfrac{1}{2}10^{-3}+\dfrac{1}{2}10^{-4}}{55\cdot 10^{-7}}\qquad$ **Odrediti x.**

$$\frac{\frac{1}{2}10^{-3}+\frac{1}{2}10^{-4}}{55\cdot 10^{-7}} = \frac{\frac{1}{2000}+\frac{1}{20000}}{\frac{55}{10000000}} =\ \text{Izvučemo gore zajednički}$$

$$\frac{\frac{1}{2000}\left(1+\frac{1}{10}\right)}{\frac{55}{10000000}} = \frac{\frac{1}{2000}\cdot\frac{11}{10}}{\frac{55}{10000000}} = \frac{\frac{11}{20000}}{\frac{55}{10000000}} =$$

$$\frac{11\cdot 10000000}{20000\cdot 55} = \frac{11\cdot 10000000}{11\cdot 100000} = \frac{10000000}{100000} = 100 = 10^2$$

sada je $10^x = 10^2$, dakle $x = 2$

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10)

a)
$$A\cdot 10^{-5} = 0{,}2\cdot 0{,}008$$
$$A\cdot 10^{-5} = 2\cdot 10^{-1}\cdot 8\cdot 10^{-3}$$
$$A\cdot 10^{-5} = 16\cdot 10^{-4}$$
$$A = \frac{16\cdot 10^{-4}}{10^{-5}}$$
$$A = 16\cdot 10^{-4-(-5)}$$
$$A = 16\cdot 10^{-4+5}$$
$$A = 16\cdot 10$$
$$A = 160$$

b)
$$B\cdot 10^{-6} = 0{,}04\cdot 0{,}006$$
$$B\cdot 10^{-6} = 4\cdot 10^{-2}\cdot 6\cdot 10^{-3}$$
$$B\cdot 10^{-6} = 24\cdot 10^{-5}$$
$$B = \frac{24\cdot 10^{-5}}{10^{-6}}$$
$$B = 24\cdot 10^{-5+6}$$
$$B = 24\cdot 10$$
$$B = 240$$

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Ovde smo koristili zapisivanje realnog broja u sistemu sa osnovnim 10. Ovo je dobra

opcija kada je broj ''glomazan''.

 Primeri:

1) Brzina svetlosti je približno $c \approx 300000000 m/s$ a mi je ''lakše'' zapisujemo
$c \approx 3\cdot10^8 m/s$, 10^8 -znači da ima 8 nula iza jedinice!!!

2) $\dfrac{1}{500000} = \dfrac{1}{5\cdot10^5} = \dfrac{1}{5}\cdot10^{-5} = \dfrac{2}{10}\cdot10^{-5} = 2\cdot10^{-1}\cdot10^{-5} = 2\cdot10^{-6}$

3) $0,000069 = 6,9\cdot10^{-5} \approx 7\cdot10^{-5}$

4) Površina zemlje je $510083000 km^2$ ali mi zapisujemo $\approx 5\cdot10^8 km^2$

11) Izračunati $\left(\dfrac{3a^{-x}}{1-a^{-x}} - \dfrac{2a^{-x}}{1+a^{-x}} - \dfrac{a^x}{a^{2x}-1}\right) : \dfrac{a^{-x}}{a^x - a^{-x}}$

$$\left(\frac{3a^{-x}}{1-a^{-x}} - \frac{2a^{-x}}{1+a^{-x}} - \frac{a^x}{a^{2x}-1}\right) : \frac{a^{-x}}{a^x - a^{-x}} =$$

$$\left(\frac{\dfrac{3}{a^x}}{1-\dfrac{1}{a^x}} - \frac{\dfrac{2}{a^x}}{1+\dfrac{1}{a^x}} - \frac{a^x}{a^{2x}-1}\right) : \frac{\dfrac{1}{a^x}}{a^x - \dfrac{1}{a^x}} =$$

$$\left(\frac{\dfrac{3}{a^x}}{\dfrac{a^x-1}{a^x}} - \frac{\dfrac{2}{a^x}}{\dfrac{a^x+1}{a^x}} - \frac{a^x}{a^{2x}-1}\right) : \frac{\dfrac{1}{a^x}}{\dfrac{a^{2x}-1}{a^x}} =$$

$$\left(\frac{3}{a^x-1} - \frac{2}{a^x+1} - \frac{a^x}{(a^x-1)(a^x+1)}\right) : \frac{1}{a^{2x}-1} =$$

$$\frac{3(a^x+1) - 2(a^x-1) - a^x}{(a^x-1)(a^x+1)} \cdot \frac{a^{2x}-1}{1} =$$

$$\frac{3a^x + 3 - 2a^x + 2 - a^x}{(a^x-1)(a^x+1)} \cdot \frac{(a^x-1)(a^x+1)}{1} =$$

$$= 3 + 2 = 5$$

12)   Izračunati $\left(\dfrac{x - x^{-2}}{x^{-2} + x^{-1} + 1} - \dfrac{x - x^{-1}}{1 + x^{-2} + 2 \cdot x^{-1}} \right) : \dfrac{1 - x^{-1}}{1 + x^{-1}}$

$$\left(\frac{x - x^{-2}}{x^{-2} + x^{-1} + 1} - \frac{x - x^{-1}}{1 + x^{-2} + 2 \cdot x^{-1}} \right) : \frac{1 - x^{-1}}{1 + x^{-1}} =$$

$$\left(\frac{x - \dfrac{1}{x^2}}{\dfrac{1}{x^2} + \dfrac{1}{x} + 1} - \frac{x - \dfrac{1}{x}}{1 + \dfrac{1}{x^2} + \dfrac{2}{x}} \right) : \frac{1 - \dfrac{1}{x}}{1 + \dfrac{1}{x}} =$$

$$\left(\frac{\dfrac{x^3 - 1}{x^2}}{\dfrac{1 + x + x^2}{x^2}} - \frac{\dfrac{x^2 - 1}{x}}{\dfrac{x^2 + 1 + 2x}{x^2}} \right) : \frac{\dfrac{x - 1}{x}}{\dfrac{x + 1}{x}} =$$

$$\left(\frac{(x-1)\,\cancel{(x^2 + x + 1)}}{\cancel{x^2 + x + 1}} - \frac{x(x-1)\,\cancel{(x+1)}}{(x+1)^{\cancel{2}}} \right) : \frac{x - 1}{x + 1} =$$

$$\left(\frac{x - 1}{1} - \frac{x(x-1)}{x + 1} \right) : \frac{x - 1}{x + 1} =$$

$$\frac{(x-1)(x+1) - x(x-1)}{x + 1} \cdot \frac{x + 1}{x - 1} =$$

$$\frac{\cancel{(x-1)}\,[(x+1) - x]}{\cancel{x+1}} \cdot \frac{\cancel{x+1}}{\cancel{x-1}} = x + 1 - x = 1$$

13) Izračunati $A = \left(\dfrac{a^n}{1-a^{-n}} + \dfrac{a^{-n}}{1+a^{-n}} \right) - \left(\dfrac{a^n}{1+a^{-n}} + \dfrac{a^{-n}}{1-a^{-n}} \right)$

$$A = \left(\frac{a^n}{1-a^{-n}} + \frac{a^{-n}}{1+a^{-n}} \right) - \left(\frac{a^n}{1+a^{-n}} + \frac{a^{-n}}{1-a^{-n}} \right)$$

$$A = \left(\frac{a^n}{1-\dfrac{1}{a^n}} + \frac{\dfrac{1}{a^n}}{1+\dfrac{1}{a^n}} \right) - \left(\frac{a^n}{1+\dfrac{1}{a^n}} + \frac{\dfrac{1}{a^n}}{1-\dfrac{1}{a^n}} \right)$$

$$A = \left(\frac{\dfrac{a^n}{1}}{\dfrac{a^n-1}{a^n}} + \frac{\dfrac{1}{a^n}}{\dfrac{a^n+1}{a^n}} \right) - \left(\frac{a^n}{\dfrac{a^n+1}{a^n}} + \frac{\dfrac{1}{a^n}}{\dfrac{a^n-1}{a^n}} \right)$$

$$A = \left(\frac{a^{2n}}{a^n-1} + \frac{1}{a^n+1} \right) - \left(\frac{a^{2n}}{a^n+1} + \frac{1}{a^n-1} \right)$$

$$A = \frac{a^{2n}(a^n+1)+1(a^n-1)}{(a^n-1)(a^n+1)} - \frac{a^{2n}(a^n-1)+1(a^n+1)}{(a^n+1)(a^n-1)}$$

$$A = \frac{a^{3n}+a^{2n}+a^n-1-(a^{3n}-a^{2n}+a^n+1)}{(a^n-1)(a^n+1)}$$

$$A = \frac{a^{3n}+a^{2n}+a^n-1-a^{3n}+a^{2n}-a^n-1}{(a^n-1)(a^n+1)}$$

$$A = \frac{2a^{2n}-2}{(a^n-1)(a^n+1)} = \frac{2(a^{2n}-1)}{(a^n-1)(a^n+1)} = \frac{2(a^n-1)(a^n+1)}{(a^n-1)(a^n+1)} = 2$$